اختبر نفسك (6)
Check yourself (6)

Mathematics الرياضيات

الصف الثاني عشر متقدم

الفصل الثالث

2025-2026

Lesson 5-6 (Integration by Substitution)

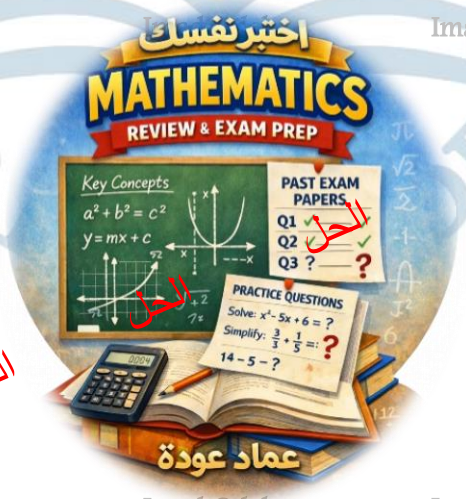
according to the previous exam

مراجعة الدرس السادس (التكامل بالتعويض)

من الوحدة الخامسة اعتمادا على

الاختبارات السابقة

الأستاذ عماد عودة



اسم الطالب: -

MCQ's الجزء الالكتروني

Q1 Which of the following can be integrated using the integration rule — أي مما يلي يمكن ايجاده باستخدام قاعدة التكامل التالي

$$\int \frac{f'(x)}{f(x)}\,dx = \ln|f(x)| + c \quad f(x) \neq 0$$

a) $\int \frac{2x}{\cos(x^2)}\,dx$

b) $\int \frac{\sin x}{\cos^2 x}\,dx$

c) $\int \frac{e^x}{\sqrt{e^x}}\,dx$

d) $\int \frac{\csc^2 x}{\cot x}\,dx$

Q2 Find — اوجد

$$\int \frac{e^x - e^{-x}}{e^x + e^{-x}}\,dx$$

a) $\ln(e^x - e^{-x}) + c$

b) $\frac{(e^x - e^{-x})^2}{2} + c$

c) $\frac{e^x + e^{-x}}{e^x - e^{-x}} + c$

d) $\ln(e^x + e^{-x}) + c$

Q3 Find — اوجد

$$\int \frac{1}{x^2}\cos\left(\frac{1}{x}\right)\,dx$$

a) $-\sin\left(\frac{1}{x^2}\right) + c$

b) $-\sin\left(\frac{1}{x}\right) + c$

c) $\sin\left(\frac{1}{x^2}\right) + c$

d) $\sin\left(\frac{1}{x}\right) + c$

Q4 Find. اوجد

$$\int 2\sin^2 x \; dx$$

a) $-2\sin x \cos x + c$

b) $2\sin x \cos x + c$

c) $\frac{\sin^3 x}{3} + c$

d) $x - \frac{\sin 2x}{2} + c$

Q5 Find اوجد

$$\int x^3\sqrt{x^4+3}\, dx$$

a) $\frac{1}{2}(x^4+3)^{3/2} + c$

b) $\frac{1}{6}x^4(x^5+3)^{3/2} + c$

c) $\frac{1}{6}(x^4+3)^{1/2} + c$

d) $\frac{1}{6}(x^4+3)^{3/2} + c$

Q6 Find اوجد

$$\int \frac{e^{\sqrt{x}}}{\sqrt{x}} dx$$

a) $\frac{1}{2e^{\sqrt{x}}} + c$

b) $\frac{2}{e^{\sqrt{x}}} + c$

c) $\frac{1}{2}e^{\sqrt{x}} + c$

d) $2e^{\sqrt{x}} + c$

Q7 Find اوجد

$$\int \sin^7 x \cos x \, dx$$

a) $\frac{-\cos^8 x}{8} + c$

b) $\frac{-\sin^8 x}{8} + c$

c) $\frac{\sin^8 x}{8} + c$

d) $7 \sin^8 x \cos x + c$

Q8

Find اوجد

$$\int_0^{\pi/2} 3 \sin^2 x \cos x \, dx$$

a) 1

b) -1

c) -3

d) 3

Q9 If

$$\int_{-2}^{6} f(x)dx = 10 \quad and \quad \int_{2}^{6} f(x)dx = 3$$

Find

$$\int_2^6 f(4-x)dx$$

a) 3

b) 6

c) 7

d) 10

Q10 If

$$\int_0^6 f(x)dx = 12$$

Find

$$\int_0^6 f(6-x)dx$$

a) **12**

b) **6**

c) **0**

d) -6

Q11 If f is continuous and $F'(x) = f(x)$ for all real numbers x

إذا كانت f دالة متصلة وكان $F'(x) = f(x)$ لجميع قيم x

Find

اوجد

$$\int_2^{10} f\left(\frac{1}{2}x\right)dx$$

a) $\frac{1}{2}[F(5) - F(1)]$

b) $\frac{1}{2}[F(10) - F(2)]$

c) $2[F(5) - F(1)]$

d) $2[F(10) - F(2)]$

FRQ الجزء الكتابي

Q1 Find

$$\int_0^1 (e^x - 2)^2 dx$$

$$\int_0^1 (e^x - 2)^2 dx = \int_0^1 (e^{2x} - 4e^x + 4)\, dx$$

$$= \left[\frac{e^{2x}}{2} - 4e^x + 4x\right]_0^1 = \frac{e^2}{2} - 4e + 4 + \frac{7}{2} = \frac{e^2}{2} - 4e + \frac{15}{2}$$

Q2 Find

$$\int x^2 e^{x^3}\, dx$$

$u = x^3 \Rightarrow du = 3x^2 dx$

$x^2 dx = \frac{1}{3} du$

$= \frac{1}{3}\int e^u du = \frac{1}{3} e^u + C = \frac{1}{3} e^{x^3} + C$

Q3 Find اوجد

$$\int \frac{3\sqrt{x}}{1+x^3}\, dx$$

في مثل هذه المسائل للتخلص من الجذر بشكل يضمن الحصول على شكل مبسط نفرض ان $u = x^{3/2}$ أي القوة الأكبر ضرب الاس الذي يقابل الجذر

$u = x^{3/2} \Rightarrow du = \frac{3}{2} x^{1/2} dx$

$3\sqrt{x} dx = 2\, du$

$$\int \frac{3\sqrt{x}}{1+x^3} dx = \int \frac{2}{1+u^2} du = 2\tan^{-1}(u) + C$$

$$= 2\tan^{-1}(x^{3/2}) + C = \boxed{2\tan^{-1}(x^{3/2}) + C}$$

Q4 Find اوجد

$$\int \frac{1}{\sqrt{x}+x}\, dx$$

$$\int \frac{1}{\sqrt{x}+x} dx = \int \frac{1}{\sqrt{x}(1+\sqrt{x})} dx$$

$u = \sqrt{x} \Rightarrow dx = 2u\, du$

$$= \int \frac{2u}{u(1+u)} du = \int \frac{2}{1+u} du = 2\ln|1+u| + C$$

$$= 2\ln(1+\sqrt{x}) + C$$

Q5 Find اوجد

$$\int \frac{(\tan^{-1} x)^2}{1+x^2} dx$$

$u = \tan^{-1} x \Rightarrow du = \frac{1}{1+x^2} dx$

$$\int u^2 du = \frac{u^3}{3} + C = \frac{(\tan^{-1} x)^3}{3} + C$$

Q6 Find اوجد

$$\int \frac{x^2}{1+x^6}\,dx = \int \frac{x^2}{1+(x^3)^2}\,dx$$

$u = x^3 \Rightarrow du = 3x^2dx$

$x^2dx = \frac{1}{3}du$

$= \frac{1}{3}\int \frac{1}{1+u^2}du = \frac{1}{3}\tan^{-1}(u) + C$

$= \frac{1}{3}\tan^{-1}(x^3) + C$

Q7 Find اوجد

$$\int x^3\sqrt{x^2-5}\;dx$$

$u = x^2 - 5 \Rightarrow du = 2xdx$

$x^2 = u + 5$

$$\int x^3\sqrt{x^2-5}\;dx = \int x^3\sqrt{u}\;\frac{du}{2x} = \frac{1}{2}\int x^2\sqrt{u}\;du$$

$= \frac{1}{2}\int (u+5)\sqrt{u}du$

$= \frac{1}{2}\int (u^{3/2} + 5u^{1/2})du$

$= \frac{1}{2}\left(\frac{2}{5}u^{5/2} + \frac{10}{3}u^{3/2}\right)$

$= \frac{1}{5}(x^2-5)^{5/2} + \frac{5}{3}(x^2-5)^{3/2} + C$

Q8 Find اوجد

$$\int \frac{dx}{e^x + e^{-x}}$$

$= \int \frac{e^x}{e^{2x}+1}dx$

$u = e^x \Rightarrow du = e^xdx$

$= \int \frac{1}{u^2+1}du = \tan^{-1}(u) + C = \tan^{-1}(e^x) + C$

Q9 Find اوجد

$$\int \frac{dx}{1+e^x}$$

Multiply top & bottom by e^{-x}

$$\int \frac{dx}{1+e^x} = \int \frac{e^{-x}}{1+e^{-x}}dx$$

$u = 1+e^{-x}$, $du = -e^{-x}\,dx$

$$\int \frac{e^{-x}}{1+e^{-x}}\,dx = \int \frac{-1}{u}\,du$$

$$= -\ln(1+e^{-x}) + C$$

Q10 If f is continuous and لتكن f دالة متصلة و

$$\int_1^8 f(x)dx = 15$$

Find

$$\int_1^2 x^2 f(x^3)dx$$

$$\int_1^2 x^2 f(x^3)dx$$

$u = x^3 \Rightarrow du = 3x^2dx$

$x = 1 \quad \rightarrow u = 1$, $x = 2$, $\quad u = 8$

$$= \frac{1}{3}\int_1^8 f(u)du = \frac{1}{3}(15) = 5$$

Q11

1) Let $F(4) = 3, F(0) = -2$ لتكن

Find اوجد

$$\int_0^2 xf(x^2)dx$$

$u = x^2 \Rightarrow du = 2xdx$

$x = 0 \quad \rightarrow u = 0$, $x = 2$, $\quad u = 4$

$$= \frac{1}{2}\int_0^4 f(u)du = \frac{1}{2}\big(F(4) - F(0)\big) = \frac{1}{2}\big(3-(-2)\big) = \frac{5}{2}$$

2) $f(1)=1, f(0)=3$

Find اوجد

$$\int_0^{\pi/2} \cos x\, f'(\sin x)\,dx$$

$u = \sin\ x$

$x = 0 \ \rightarrow u = 0\ , x = \pi/2\ , \quad u = 1$

$$= \int_0^1 f'(u)du = f(1) - f(0) = 3 - 1 = 2$$

3) Let $F(2)=-3, F(0)=2$ لتكن

Find اوجد

$$\int_0^4 \frac{f(\sqrt{x})}{\sqrt{x}}\,dx$$

$u = \sqrt{x} \Rightarrow dx = 2udu$

$x = 0 \ \rightarrow u = 0\ , x = 4\ , \quad u = 2$

$$= \int_0^2 \frac{F(u)}{u}(2udu) = 2\int_0^2 F(u)du = F(2) - F(0) = -3 - 2 = -5$$

BEST WISHES TO ALL

اطيب التمنيات للجميع